Can tetraneutron exist from theoretical point of view?

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Abstract

A theoretical possibility is shown for the bound state of a tetraneutron to exist in the case of the proposed neutron-neutron potentials in the singlet state with two attractive wells separated by a repulsive barrier. The anomalous behaviours are revealed for the calculated size, density distribution, and pair correlation functions of a hypothetical tetraneutron.

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1 Introduction

The mysterious fact of the experimental registration of tetraneutrons [1, 2] in a reaction with loosely bound radioactive ^{14}Be renewed the attention to theoretical attempts of understanding the problem of hypothetical bound neutron systems. The experiment contradicts the rather old estimates showing the impossibility to form a bound state of a few neutrons interacting by the standard nuclear forces (see review [3]). Recent attempts to study this problem using more modern methods of calculation of four-particle systems [4, 5, 6] also indicate the impossibility of binding the four-neutron system without adding some exotic many-particle interaction potentials. Moreover, the absence of resonances in a four-neutron system with standard potentials was shown in [7]. The experimental search for resonances in ^4n and ^3n systems [8] using other nuclear reactions had also no success.

In the present paper, we study the theoretical problem of the possible existence of a tetraneutron by developing the precise methods of calculation of loosely bound states of four Fermi particles. We propose a special idea of constructing the neutron-neutron potentials allowing the bound tetraneutron to exist and simultaneously describing the standard low-energy neutron-neutron data.

2 Basic equations

To study the properties of the four-neutron system in the state with zero spin ($S = 0$) and orbital moment ($L = 0$) under assumption of the central pairwise neutron-neutron interaction potentials,

we solve the following Schrödinger equation for one spatial component of the wave function:

$$\{\sum_{i=1}^{4}\frac{\mathbf{p}_i^2}{2m}+\frac{1}{2}\sum_{i>j=1}^{4}\left(V_s^{+}(r_{ij})+V_t^{-}(r_{ij})\right)+\frac{1}{2}\sum_{(ij)\neq(14),(23)}(-1)^{i+j}\left(V_s^{+}(r_{ij})-V_t^{-}(r_{ij})\right) \tag{1}$$
$$-\frac{1}{2}\sum_{(ij)\neq(12),(34)}(-1)^{i+j}\left(V_s^{+}(r_{ij})-V_t^{-}(r_{ij})\right)\hat{P}_{23}\}\Phi=E\Phi.$$

The total antisymmetric wave function of the four-neutron system is expressed in terms of the corresponding spin and spatial components as

$$\Psi^a(1,2,3,4)=\frac{1}{\sqrt{2}}\left(\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\xi''-\Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\xi'\right), \tag{2}$$

where the antisymmetric Φ' and symmetric Φ'' (with respect to permutations $(1\rightleftarrows 2)$ and $(3\rightleftarrows 4)$) spatial components are

$$\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\equiv\Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4), \tag{3}$$
$$\Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\equiv\frac{1}{\sqrt{3}}\left(2\Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)-\Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right),$$

which corresponds to the Young scheme [2,2]. In Eq. (1), $\hat{P}_{23}$ is the permutation operator of spatial coordinates, $V_s^{+}(r_{ij})$ and $V_t^{-}(r_{ij})$ are, respectively, the singlet interaction potential in even states and the triplet one in odd states. The bound states of four neutrons are studied by solving the Schrödinger equation (1) for various nuclear potentials taken in the form of a superposition of Gaussian functions, by using the well-known variational method with the translationally invariant Gaussian basis antisymmetrized with respect to the permutations of particles $(1\rightleftarrows 2)$ and $(3\rightleftarrows 4)$,

$$\Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=\hat{A}\sum_{k=1}^{N}C_k\exp\left(-\sum_{i>j=1}^{4}u_{ij}^k r_{ij}^2\right), \tag{4}$$

where N is the basis dimension, and $r_{ij}\equiv|\mathbf{r}_i-\mathbf{r}_j|$. This basis and the special schemes necessary to optimize the nonlinear variational parameters u_{ij}^k enable us to carry on the calculations of loosely bound states with desired high accuracy.

3 Spinless interaction model

First, consider the simplest case of potentials independent of spin (i.e. $V_t^{-}(r_{ij})=V_s^{+}(r_{ij})$) where the overestimated attraction in the triplet state may only promote the binding of the four-neutron system. We study the ^4n bound state appearance conditions varying the coupling constant of potentials of different forms (further, we use the dimensionless units: $V(r)=\frac{\hbar^2}{mr_0^2}U(\frac{r}{r_0})\equiv\frac{\hbar^2}{mr_0^2}gu(r)$, where r_0 is the radius of interaction, and g is the coupling constant; $\hbar^2/m=41.4425$ MeV·fm^2 for neutrons). For the potential with one Gaussian function, $U(r)=-g\exp(-r^2)$, a bound tetraneutron ^4n exists below the decay threshold $(4\rightarrow 2+2)$ only for $g\geq g_{cr}(4)=3.911$, which is 1.46 times greater than the critical two-particle coupling constant $g_{cr}(2)=2.684$. We notice that the reliable calculations need basis (4) to be about 150 functions with the optimization of nonlinear parameters. Note that

a trineutron, for the same potential, can be bound below the decay threshold ($3 \to 2+1$) only for $g \geq 3.3 g_{cr}(2)$. From the qualitative point of view, the similar conditions of the tetraneutron bound state appearance take place for other traditional purely attractive potentials. Thus, under the condition that a dineutron is unbound, there is no possibility, because of the Pauli principle, for a tetraneutron (nothing to say of a trineutron) to form a bound state with traditional attractive potentials. An analogous conclusion can be drawn in the case of common potentials with repulsion at short distances between neutrons. For example, for the widely used Volkov potential, one has $k \equiv g_{cr}(4)/g_{cr}(2) = 1.44$. Moreover, the attempt to find a better ratio k by varying the parameters of the two-component potentials with attraction and short-range repulsion led us only to a potential $U(r) = g\,(1.5\exp(-(r/0.9)^2) - \exp(-r^2))$ giving rise to k about 1.27. We can assume that it is impossible to form a bound system ^4n also for other standard interaction potentials with attraction and short-range repulsion if ^2n is unbound, which is in agreement with the recent calculations [4, 5, 6].

In principle, a possibility for a bound tetraneutron to exist, under the condition of an unbound ^2n, can be realized with some exotic pairwise interaction potentials having two regions of attraction separated by a repulsive barrier. An external attractive potential well of greater radius is necessary, first of all, to fit the experimental low-energy two-neutron scattering parameters in the singlet state, and it has to be in the typical range of nuclear forces of about or greater 1.5 fm. Fitting the singlet interaction potential, we use the following low-energy neutron-neutron scattering parameters: the scattering length $a_{s(nn)} = -18.9$ fm and effective radius $r_{0s(nn)} = 2.75$ fm. The internal attractive potential well of smaller radius is important for binding the ^4n system, while the repulsive barrier between the attractive wells makes the two regimes of attraction somewhat independent. In a tetraneutron, the number of pairs of particles in the singlet state, as well as that in the triplet one, equals three, the Pauli principle reveals itself only in the triplet state, and the internal potential well acting in the singlet state plays the main role in binding the ^4n system. A class of potentials with two attractive wells of different radii, which give rise to the bound state of ^4n under the assumption $V_t^-(r_{ij}) = V_s^+(r_{ij})$, is rather wide. We have a number of potentials in the form of a superposition of three or four Gaussian functions. One of the optimal variant is the four-component neutron-neutron singlet potential

$$U_s^+(r) = g\left\{0.43\exp\left(-\left(r/0.6\right)^2\right) - \exp\left(-r^2\right) + 1.085\exp\left(-\left(r/1.3\right)^2\right) - 0.42\exp\left(-\left(r/1.5\right)^2\right)\right\}, \tag{5}$$

where the distance is measured in units of $r_0 = 0.488519$ fm. At $g = g_{exper} = 322.40$, potential (5) reproduces the experimental values of $a_{s(nn)}$, $r_{0s(nn)}$, and the commonly used recommended singlet neutron-neutron phase shift up to the energies $E_{\rm lab} \approx 80$ MeV. Note that there are no direct measurements of the neutron-neutron phase shifts.

Fig. 1 shows the dependence of the tetraneutron energy on the coupling constant g of potential (5) in the "spinless" case $U_t^-(r_{ij}) = U_s^+(r_{ij})$. Note that the decay threshold of ^4n into $2+2$ (as well as the decay threshold of ^3n into $2+1$) as a function of the coupling constant has two regimes of behaviour. The first regime of a rather weak binding of ^2n at $g \longrightarrow g_{cr}(2)$ takes place due to the presence of the attraction of greater radius in potential (5), and the second one with the almost linear dependence of the threshold in a wide range of coupling constants is present due to the attraction of smaller radius. A repulsive barrier between the attractive wells contributes to the sharpness of changing the two regimes of the threshold behaviour. Note also that the excited two-particle S-state lies anomalously close to the ground state at $g^*_{cr}(2)/g_{cr}(2) = 1.12$, which is caused to a great extent by the presence of two almost independent attractive wells in potential (5). It is essentially important that, in variational calculations, the ^4n system is bound already at $g \geq g_{cr}(4) = 315.2 = 0.954 g_{cr}(2)$, where ^2n is still unbound ($g \leq g_{cr}(2) = 330.42$). Moreover, at the coupling constant $g = g_{exper} = 322.40 = 0.976 g_{cr}(2)$, where potential (5) reproduces the experimental low-energy neutron-neutron

parameters, the ^4n system is already bound. At the same time, a trineutron with the considered potential is not allowed to be bound since the ratio $g_{cr}(3)/g_{cr}(2) = 1.008$ is greater than 1, although being close to it. Notice the fact that the ^4n energy dependence on g looks like almost a straight line parallel to the energy threshold $(2+2)$ dependence in a wide interval of coupling constants ($g \gtrsim 1.1$), and this line is rather close to the $(2+2)$ threshold. This fact indicates that, in this region of g, a tetraneutron exists due to the presence of the internal potential well of smaller radius, and the ^4n state is of the two-dineutron cluster nature. A similar consideration concerns ^3n as well: in a wide interval of coupling constants, it is the cluster state $(2+1)$ with the essential role of the attractive well of smaller radius. This is confirmed also by the approximate relation $E_{^4n} - 2E_{^2n} \approx 2\,(E_{^3n} - E_{^2n})$. By the way, we constructed some other variants of potentials $U_s^+(r)$, for example,

$$U_s^+(r) = g\left\{0.315\exp\left(-\left(r/0.5\right)^2\right) - \exp\left(-r^2\right) + 1.278\exp\left(-\left(r/1.31\right)^2\right) - 0.54\exp\left(-\left(r/1.5\right)^2\right)\right\},$$

which yields $g_{cr}(4)/g_{cr}(2) = 0.9525$ for ^4n to exist and reproduces the low-energy parameters of $n-n$ scattering at $g_{exper}/g_{cr}(2) = 0.9935$ ($g_{cr}(2) = 187.5$, and interaction radius $r_0 = 1.2608$ fm). Even a trineutron could exist for this potential upon the unbound ^2n due to the ratio $g_{cr}(3)/g_{cr}(2) = 0.9564$ being less than unity, but the more strict condition $g_{cr}(3) < g_{exper}$ is not valid. In addition, the $n-n$ singlet phase shift for this potential becomes too large already at rather low energies. We have found no variant of the potential obeying the condition $g_{cr}(3) < g_{exper}$ simultaneously with giving a reasonable phase shift at low energies for ^3n to exist in a bound state.

It should be noted that it is necessary to carry on variational calculations with special schemes of optimization of basis (4) in order to obtain the above results for ^4n with reliable accuracy. In particular, we used about 220 functions with the optimization of the basis for this purpose. This is caused by both the complicated antisymmetrized four-neutron wave function of the near-threshold state and the potential containing essentially different components.

4 Realistic case

Now consider more realistic models of the triplet interaction potential $U_t^-(r)$, when the conditions for the existence of the bound state of a tetraneutron are somewhat less appropriate. If we put $U_t^-(r)$ to be zero in Eq. (1), we get an unbound tetraneutron within the proposed class of singlet potentials, because only a half of 6 pairs interacts in this case. Thus, it is necessary to have some additional attraction in odd orbital states to bind ^4n. On the other hand, the commonly recommended phase shifts of the scattering in odd orbital states are rather negative corresponding to the effective repulsion. It appears that there exists a class of triplet potentials which together with the singlet potential (5) can bind the ^4n system and are repulsive with the exception of the typical nuclear distances of about $1.5-2$ fm, where they reveal some attraction correlated with the external attractive potential well of the singlet potential. Such a potential (in the same dimensionless units, as potential (5)) may have the form

$$U_t^-(r) = g_t\left\{2.212\exp\left(-\left(r/2\right)^2\right) - 2.334\exp\left(-\left(r/3\right)^2\right) + \exp\left(-\left(r/4\right)^2\right)\right\} \qquad (6)$$

with $g_t = 14$. This potential has negative phase shift in the P-state, although with non-monotone dependence. Potential (6) together with the singlet one (5) result in the bound state of a tetraneutron with the binding energy $B(^4n) \gtrsim 0.5$ MeV (this value is the variational estimation with the use of about 400 Gaussian functions). A more accurate calculation needs much greater efforts mainly because of the complicated structure of potentials and the many-component antisymmetrized wave function of four particles. In addition, the ultimate result for the binding energy is a few orders

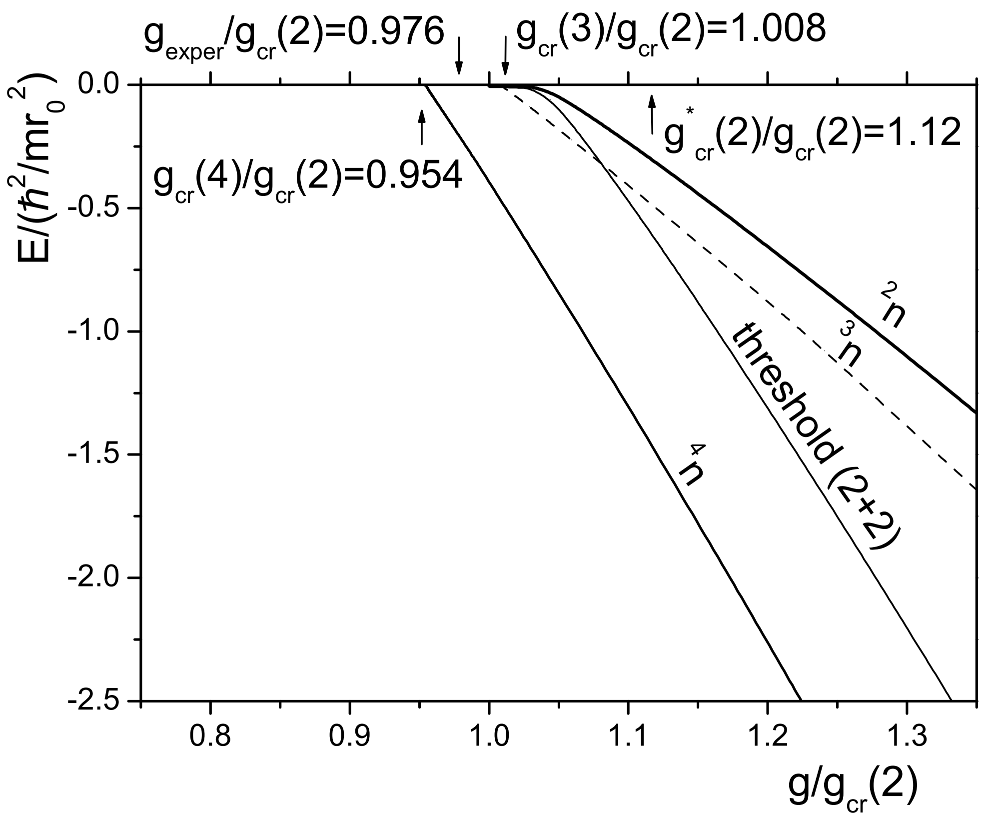

Figure 1: Tetraneutron energy dependence on the coupling constant of potential (5) acting both in the singlet and triplet states ($r_0 = 0.488519$ fm is the radius of the interaction potential (5)).

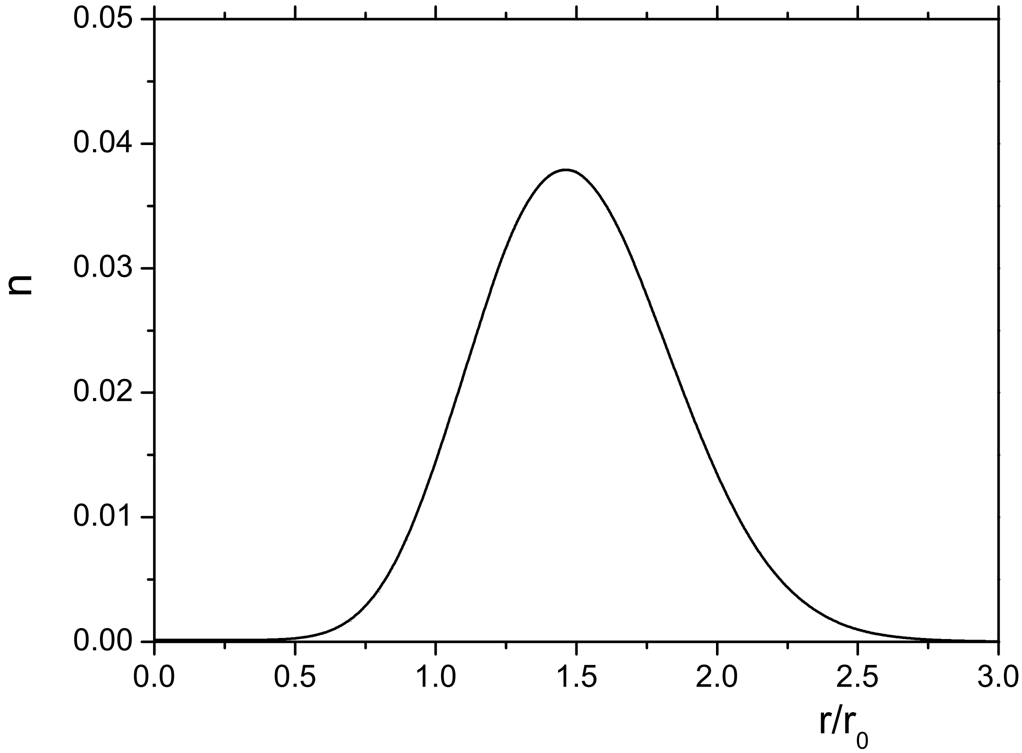

Figure 2: Profile of the density distribution $n(r)$ of neutrons of the hypothetical tetraneutron (r_0 is the same as in Fig. 1).

of magnitude lesser than the contributions of the kinetic or potential energies calculated separately with rather good accuracy, and these contributions almost cancel each other having opposite signs. The proposed phenomenological potentials are constructed only to demonstrate the possibility for a tetraneutron to exist in a bound state. Moreover, one can easily change the binding energy of ^4n in a wide range (from zero to dozens of MeV) by changing slightly potential (6) or potential (5). There are some reasons to assume that if these potentials with repulsive barriers are changed in such a way that they should not allow a tetraneutron to be bound, they may result in resonances in the system of four neutrons.

Consider the main structure functions of the hypothetical tetraneutron. Note that the structure functions can be calculated much more accurately using basis (4) of a lesser dimension than that used in the calculation of the energy. Fig. 2 presents the one-particle density distribution of ^4n (normalized as $\int n(r)d\mathbf{r} = 1$) versus the dimensionless distance, for potentials (5), (6). Due to the Pauli principle, the density distribution has essential minimum at short distances, i.e. the tetraneutron is a "bubble" system with the almost Gaussian near-surface distribution of neutrons. The tetraneutron has anomalously small (in nuclear scale) r.m.s., $\langle r^2 \rangle^{1/2} = 1.704 r_0 = 0.83$ fm, which is caused by the attraction well of smaller radius in potential (5). Changing the singlet potential (5), one can increase the above value mainly due to an increase of the radius r_0. But, in any case, the size of a tetraneutron, in spite of its extremely small binding energy, will be less or about the size of an α-particle.

Fig. 3 depicts the singlet $g_{2s}(r)$ and triplet $g_{2t}(r)$ pair correlation functions, which reflect, to a great extent, the behaviour of the corresponding potentials. The singlet correlation function $g_{2s}(r)$

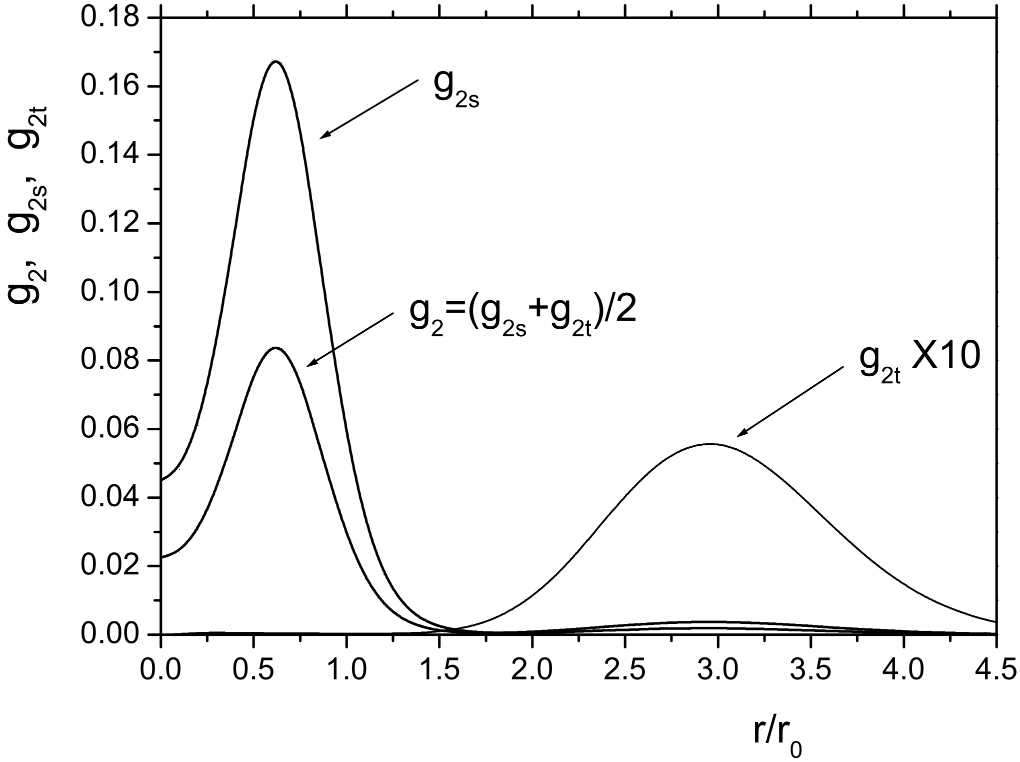

Figure 3: Profiles of the singlet $g_{2s}(r)$, triplet $g_{2t}(r)$, and total $g_2(r) = \frac{1}{2}\left(g_{2s}(r) + g_{2t}(r)\right)$ pair correlation functions of the hypothetical bound ^4n system (r_0 is the same as in Fig. 1).

has significant maximum in the region of internal short-range attraction of the singlet potential, since the Pauli principle does not reveal itself in the singlet state. Some decrease of $g_{2s}(r)$ at very short distances is caused by the presence of short-range repulsion in potential (5), and it should not be present if the repulsion were absent. The secondary maximum is present in $g_{2s}(r)$ due to the existence of the external attractive potential well of $V_s^+(r)$. In the triplet state, the repulsion at short distances makes a small contribution into the energy because of the Pauli principle, and the maximum of $g_{2t}(r)$ is located in the attractive area of the triplet potential. That is why, the contribution of the triplet potential to the energy of ^4n is negative, and a tetraneutron could not be bound without the contribution of this comparatively small effective attraction. The short-range attraction in the singlet state plays the main role in binding the ^4n system. This is confirmed by calculations of the average singlet and triplet potential energy contributions,

$$\langle V\rangle = 3\left\{\int V_s^+(r)g_{2s}(r)d\mathbf{r} + \int V_t^-(r)g_{2t}(r)d\mathbf{r}\right\} \equiv \left\langle V_s^+\right\rangle + \left\langle V_t^-\right\rangle,$$

where $\left\langle V_s^+\right\rangle = -1296.7$ MeV and $\left\langle V_t^-\right\rangle = -158.7$ MeV, which together with the kinetic energy $\left\langle \hat{K}\right\rangle = 1454.9$ MeV result in the negative energy of the system $E_{^4n} \lesssim -0.5$ MeV indicated above.

The total correlation function $g_2(r) = \frac{1}{2}\left(g_{2s}(r) + g_{2t}(r)\right)$ reflects the average neutron pair correlations and has the main maximum at short distances and the secondary one in the region of attraction in the triplet state (where the singlet potential also has an external attractive well).

5 Conclusions

To summarize, we note the following. 1) A tetraneutron can exist in the bound state if one assumes that the interaction potential in the $n-n$ singlet state has two attractive wells separated by a repulsive barrier. Unfortunately, as a result, we get an anomalously high maximum in the singlet scattering phase shift $\delta_s \approx 160^0$ at the energies of neutrons of the order of $100 - 150$ MeV. The problem of constructing the potential, which binds ^4n and gives no anomalous maximum mentioned above, is to be further studied. Maybe, a combination of pairwise potentials with some small intercluster ones can improve the situation. The nature of the assumed exotic short-range attraction is to be discussed as well. 2) Strange though it may seem, few-nucleon systems are essentially underbound with the $n-n$ potentials (5), (6) used together with the standard $n-p$ ones. In particular, with the Minnesota potential used as the $n-p$ interaction, one has the binding energies of about 6.2 MeV for ^{3}H, and about 23.4 MeV for ^{4}He. The calculated few-nucleon values may be in agreement with the experimental data under the condition that the fitting of the potentials is carried out taking into account the concordance of the regimes of attraction of all the potentials. 3) The hypothetical tetraneutron has abnormal small size and, at the same time, small binding energy, and this may serve as an additional criterion for the identification of such systems. It is interesting to study the probability of the tetraneutron "presence" in ^{14}Be nuclei. 4) Potentials (5) and (6) satisfy the saturation conditions necessary for the stability of the neutron matter. Interesting and nontrivial questions arise concerning heavier multineutron systems with such potentials. In particular, a system of 8 neutrons can be a more stable system than ^4n both because of the magic number of particles, and because the former may be like two tetraneutron clusters.

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